

RESEARCH ARTICLE

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# Performance comparison of first-order conditional estimation with interaction and Bayesian estimation methods for estimating the population parameters and its distribution from data sets with a low number of subjects

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## Abstract

**Background:** Exploratory preclinical, as well as clinical trials, may involve a small number of patients, making it difficult to calculate and analyze the pharmacokinetic (PK) parameters, especially if the PK parameters show very high inter-individual variability (IIV). In this study, the performance of a classical first-order conditional estimation with interaction (FOCE-I) and expectation maximization (EM)-based Markov chain Monte Carlo Bayesian (BAYES) estimation methods were compared for estimating the population parameters and its distribution from data sets having a low number of subjects.

**Methods:** In this study, 100 data sets were simulated with eight sampling points for each subject and with six different levels of IIV (5%, 10%, 20%, 30%, 50%, and 80%) in their PK parameter distribution. A stochastic simulation and estimation (SSE) study was performed to simultaneously simulate data sets and estimate the parameters using four different methods: FOCE-I only, BAYES(C) (FOCE-I and BAYES composite method), BAYES(F) (BAYES with all true initial parameters and fixed  $\omega^2$ ), and BAYES only. Relative root mean squared error (rRMSE) and relative estimation error (REE) were used to analyze the differences between true and estimated values. A case study was performed with a clinical data of theophylline available in NONMEM distribution media. NONMEM software assisted by Pirana, PsN, and Xpose was used to estimate population PK parameters, and R program was used to analyze and plot the results.

**Results:** The rRMSE and REE values of all parameter (fixed effect and random effect) estimates showed that all four methods performed equally at the lower IIV levels, while the FOCE-I method performed better than other EM-based methods at higher IIV levels (greater than 30%). In general, estimates of random-effect parameters showed significant bias and imprecision, irrespective of the estimation method used and the level of IIV. Similar performance of the estimation methods was observed with theophylline dataset.

(Continued on next page)

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**Conclusions:** The classical FOCE-I method appeared to estimate the PK parameters more reliably than the BAYES method when using a simple model and data containing only a few subjects. EM-based estimation methods can be considered for adapting to the specific needs of a modeling project at later steps of modeling.

**Keywords:** Estimation methods, Few subjects, First-order conditional estimation with interaction, Markov chain Monte Carlo Bayesian, NONMEM,

## Background

Exploratory preclinical (as well as clinical) trials may involve a low number of subjects (around 6 subjects). This is because in the early stages of drug development, statistical approaches are difficult to apply, potentially leading to bias when predicting population mean and distribution of parameters and/or all sources of variability. In addition, different aspects of the study design are not considered when calculating the number of subjects. As a result, it can be difficult to calculate and analyze the pharmacokinetic (PK) parameters, especially if the PK parameters show very high inter-individual variability (IIV).

Population analysis is a set of statistical techniques that can be used to study the average response (clinically measured event of any biomarker) in a population, as well as the IIVs in responses arising from different sources [1]. NONMEM is the gold standard software for population analysis that allows for mixed-effect modeling of PK/pharmacodynamic data while accounting for both unexplained inter-subject, inter-occasion, and residual variability (random effects), as well as measured concomitant effects (fixed effects). It can also be useful for analyzing data obtained from a low number of subjects involved in a study [2]. A list of estimation methods is available in NONMEM, including classical estimation methods [first-order conditional estimation with interaction (FOCE) and second-order approximation (LAPLACE)] and maximum likelihood expectation maximization (EM)-based estimation methods [iterative two-stage (ITS), important sampling EM (IMP), important sampling EM assisted by mode a posteriori (IMPMP), stochastic approximation expectation maximization (SAEM), and Markov chain Monte Carlo Bayesian (BAYES)]. Therefore, it is important to understand the performance of different approach-based methods for handling data with a low number of subjects.

Classical estimation methods like FOCE-I, including FO, FOCE and Laplace, approximate the likelihood by taking Laplace transformation and Taylor linearization [3]. These methods are known to perform well when models structure are simple and low in dimension. Here, the model with higher number of random-effect parameters (IIVs) are referred as of high dimensions. Furthermore, the classical estimation methods known to provide highly reproducible values, and short run-times

for simple PK models [4]. However, these linearization methods fail to converge and estimate parameters precisely with significant bias with increase in model complexity. The EM based methods calculate the exact likelihood (with approximation) by sampling and summing through the probability density function space, which is theoretically expected to approach the true likelihood as the sampling reaches infinity. It is due to this sampling step EM based methods have longer run-time compared to the classical methods for simple PK models [5]. In case of complex PK/PD problems, EM based methods are faster than FOCE-I due to their efficient maximization step [4].

Some previous studies have compared available estimation methods with different objectives, identifying various desirable traits of estimation methods. The most desirable property of a given estimation method is its precision and accuracy as they are the basis of the reliability of the obtained estimates. Other expected features of the estimation methods are low sensitivity to priors and short runtime. However, no previous study has compared estimation methods for estimating population PK parameters from a small number of subjects. Therefore, the objective of this study was to compare precision and accuracy of estimation methods for estimating population mean and distribution of PK parameters from a small number of subjects and explore options to minimize bias with a classical method and a maximum likelihood EM-based method.

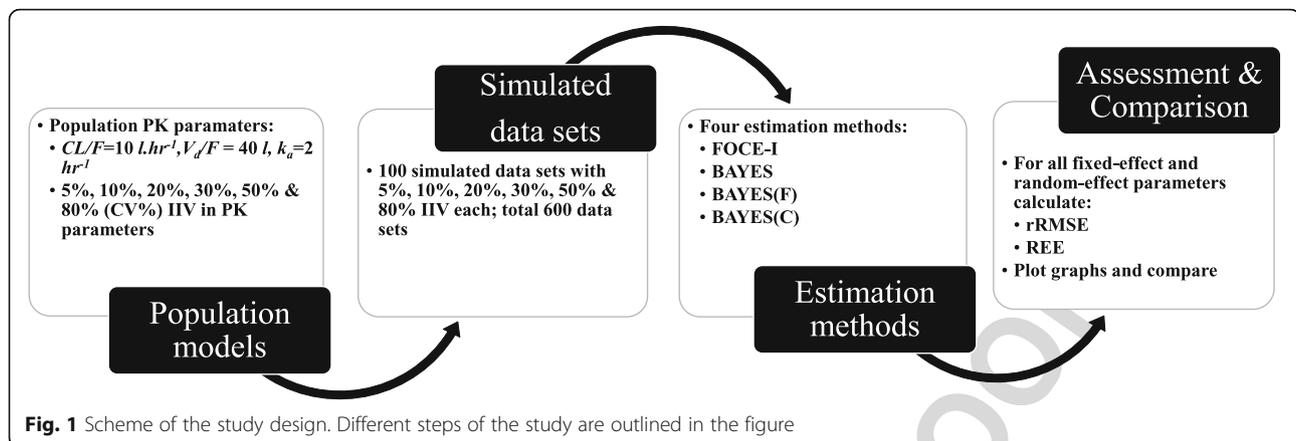
## Methods

An outline of this study is provided in Fig. 1; details are given in the following subsections. In this study, 100 data sets were simulated with eight sampling points for each subject and with six different levels of IIV (5%, 10%, 20%, 30%, 50%, and 80%) in their PK parameter distribution. The main reason for creating data sets was to describe close to real situations and minimize potential data set-dependent bias.

### Stochastic simulations and estimations

A stochastic simulation and estimation (SSE) study was performed using a one-compartment PK model. The estimation options in the model were varied to assess the performance of a classical estimation method –

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f1.1  
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128 FOCE with the interaction option (FOCE-I), which al- 152  
 129 lows for interaction between IIV( $\eta$ ) and residual variabi- 153  
 130 lity( $\epsilon$ ), and an EM-based estimation method – BAYES 154  
 131 estimation method in NONMEM version 7.3.0 [6] 155  
 132 assisted by Pirana (ver. 2.9.0), PsN (ver. 4.2.0), and Xpose 156  
 133 (ver. 4.4.1) [7]. For statistical analysis of the results and 157  
 134 generating different plots of the results, R (ver. 3.1.3) 158  
 135 program was used [8]. 159

### 136 Population model and simulated data sets

137 The population model, specifically a one-compartment 160  
 138 open model with first-order absorption and elimination 161  
 139 rate constants, was used for simulation and estimation. 162  
 140 The model consisted of three systematic PK parameters 163  
 141 as fixed effects describing the absorption rate constant 164  
 142 ( $K_a$ ), apparent volume of distribution ( $V_d/F$ ), and appar- 165  
 143 ent clearance ( $CL/F$ ), two random-effect parameters ( $\eta$ ) 166  
 144 describing the IIV on  $V_d/F$  and  $CL/F$  [Eqs. (1, 2 and 3)], 167  
 145 and a proportional error ( $\epsilon$ ) model (Eq. 4): 168

$$146 \quad K_a = \theta_{K_a}, \quad (1)$$

$$147 \quad V_d/F = \theta_{V_d/F} \cdot e^{\eta_{V_d/F}}, \quad (2)$$

$$148 \quad CL/F = \theta_{CL/F} \cdot e^{\eta_{CL/F}}, \quad (3)$$

$$149 \quad C_{ij} = C_{pred,ij} (1 + \epsilon_{ij}), \quad (4)$$

150 where  $C_{ij}$  indicates the  $j$ -th observations of  $i$ -th individual, 170  
 151  $C_{pred, ij}$  indicates the model-predicted  $C_{ij}$ , and  $\epsilon_{ij}$  indicates 171  
 152 the proportional residual error. 172

153 The following equations [Eqs. (5) and (6)] describe the 173  
 154 rate of change in drug amount in a one-compartment 174  
 155 system: 175

$$156 \quad \frac{dA_d}{dt} = -K_a A_d, \quad (5)$$

$$157 \quad \frac{dA_c}{dt} = K_a A_d - \frac{CL/F}{V_d/F} A_c, \quad (6)$$

158 where  $A_d$  and  $A_c$  are the drug amounts in the depot and 176

central compartments, respectively, and  $t$  denotes the 152  
 153 time. 154

The data set used for simulation consisted of six indi- 154  
 155 viduals with eight sampling points within 24 h after dosing 156  
 157 for each individual. The population mean of PK 158  
 159 parameters were assumed to be 2 L/h, 40 L, and 10 L/h 160  
 161 for  $K_a$ ,  $V_d/F$ , and  $CL/F$ , respectively and their IIV levels 162  
 163 (variance parameter  $\omega^2$ ) were assumed to be 5%, 10%, 164  
 165 20%, 30%, 50%, and 80% coefficient of variance (CV%) 166  
 167 (Eq. 7). 168

$$169 \quad CV(\%) = \sqrt{e^{\omega^2} - 1} \times 100\%. \quad (7)$$

Data sets were simulated 100 times for each level of 169  
 170 IIV (total of 600 data sets) and tested to compare esti- 171  
 172 mation performance in NONMEM. 173

### 174 Estimation methods

The population model was fitted to each of the simu- 174  
 175 lated data sets using estimation methods with different 176  
 176 estimation options and open or fixed  $\omega^2$  values, as sum- 177  
 177 marized in Table 1. 178

The FOCE-I method is a classical estimation method 179  
 180 that is applied by most users and has a short run-time 181  
 182 for estimation of population mean and distribution for 182  
 183 simple models [9, 10]. The BAYES method is a newly 183  
 184 introduced method in NONMEM and is more suitable 184  
 185 for estimation of population mean and distribution for 185  
 186 complex PK/PD models [10]. In this study, the other 186  
 187 estimation methods such as ITS, IMP, IMPMAP, and 187  
 188 SAEM were not tested because these methods were 188  
 189 expected to perform similar or below the performance 189  
 190 of BAYES as these methods are based on EM algorithms. 190  
 191 EM algorithms consist of an expectation (E) and a 191  
 192 maximization step (M), where these methods differed in 192  
 193 the way step E was performed, which involves the 193  
 194 approximation of likelihood. Additionally, the BAYES 194  
 195 method creates a large sample of probable parameters, 195

T1

**Table 1** Estimation methods and their conditions for initial parameters and estimation options

Methods	FOCE-I	BAYES(C)	BAYES(F)	BAYES
Conditions	First-order conditional estimation with interaction	FOCE-I and BAYES composite method	BAYES with $\omega^2$ value fixed to true value	Markov chain Monte Carlo Bayesian
Initial parameters	THETAs & OMEGAs: Open true values	THETAs & OMEGAs: Open true values	THETAs: Open true values OMEGAs: Fixed true values	THETAs & OMEGAs: Open true values
Estimation options	SIG = 3	For FOCE-I, SIG = 3 For BAYES, CTYPE = 3 NBURN = 4000 NITER = 10,000 SIGL = 8 NSIG = 2	CTYPE = 3 NBURN = 4000 NITER = 10,000 SIGL = 8 NSIG = 2	CTYPE = 3 NBURN = 4000 NITER = 10,000 SIGL = 8 NSIG = 2

186 unlike other EM-based methods that attempt to obtain a  
187 single “most likely” set of estimates.

188 In this study, true parameter values, i.e., the parameter  
189 values used in the simulation step, were established as  
190 initial estimates in all estimation methods. In NON-  
191 MEM, convergence criteria for a FOCE-I are based only  
192 on the parameter estimation gradient and are tested by  
193 default. The number of significant digits for the estima-  
194 tion of each parameter was set to three (SIG = 3) for the  
195 FOCE-I method. In the BAYES estimation method, the  
196 convergence test type was set to 3 (CTYPE = 3), where  
197 changes in objective function value, THETAs, OMEGAs,  
198 and SIGMAs, are accessed. The number of significant  
199 digits to which the objective function was evaluated was  
200 set to 8 (SIGL = 8). In the BAYES methods, the max-  
201 imum number of iterations for which to perform the  
202 burn-in phase was set to 4000 (NBURN = 4000), and the  
203 number of iterations for which to perform the stationary  
204 distribution for BAYES analysis was set to 10,000  
205 (NITER = 10,000), both of which are default values in  
206 NONMEM. The former option ensured that all paramet-  
207 ers and objective functions did not appear to move in a  
208 specific direction, but appeared to instead move around  
209 a stationary region, and the latter provides a large set  
210 (10,000) of likely population parameters.

### 211 Assessment and comparison of estimation methods

212 The estimation methods were assessed by relative root  
213 mean squared error (rRMSE) and relative estimation  
214 error (REE) for fixed-effect as well as random-effect pa-  
215 rameters to calculate and visualize the magnitude of dif-  
216 ferences between the true value and the estimated value.  
217 The rRMSE [Eq. (8)] provides a combined measure of  
218 bias and precision.

$$rRMSE = \sqrt{\frac{\sum (P_{est} - P_{true})^2}{P_{true}^2 n^2}} \quad (8)$$

219 where  $P_{est}$  is the estimated parameter value,  $P_{est}$  is the

220 true parameter values used at the simulation step, and  $n$   
221 is the number of simulations for each set of  $P_{true}$  ( $n =$   
222 100).

223 REE was calculated [Eq. (9)] and plotted as box plots;  
224 the plot represents the relative bias by the median of the  
225 REE values and precision by distribution of REE about  
226 zero.

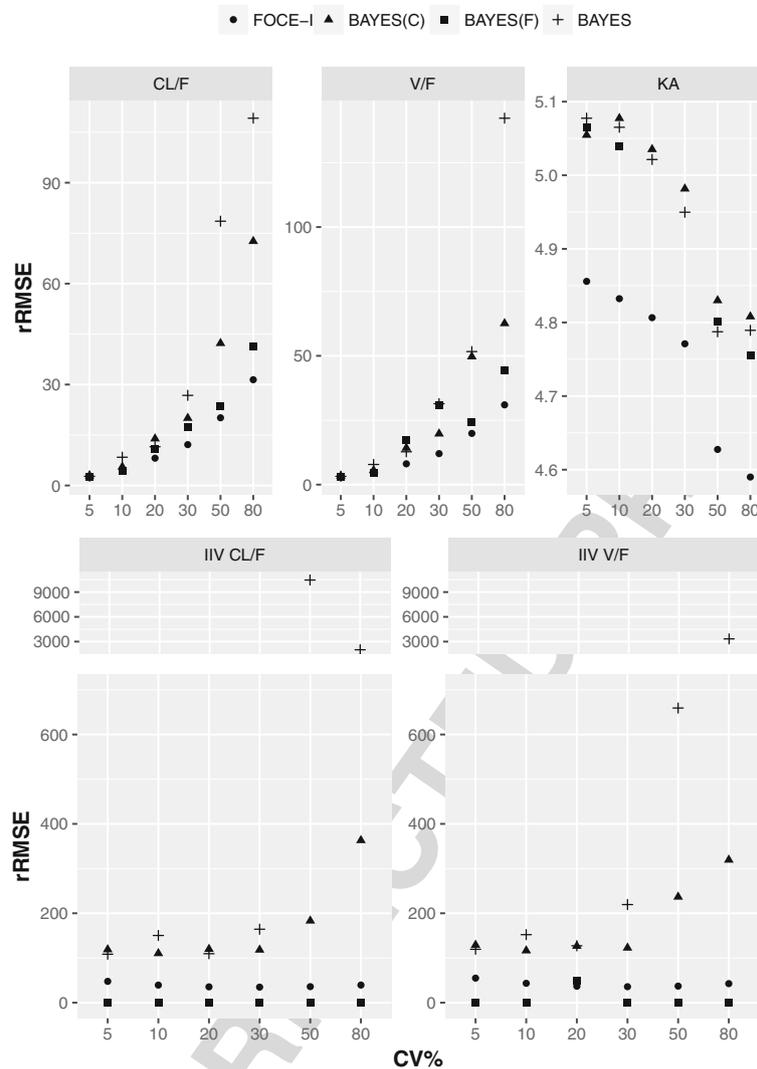
$$REE = \sqrt{\frac{P_{est} - P_{true}}{P_{true}}} \quad (9)$$

### 227 Case study

228 The THEO data set available in the NONMEM distribu-  
229 tion media was used as a case study. The estimated PK  
230 parameters and IIV from the final model fitted to the  
231 THEO data (called THEO model hereafter) was consid-  
232 ered to be the population (true) mean values for PK pa-  
233 rameters and IIV. SSE was performed using the THEO  
234 model, where 100 data sets were simulated from the  
235 model with six individuals in each data sets and four dif-  
236 ferent estimation methods were used, listed in Table 1,  
237 to estimate the PK parameters and their IIV from the  
238 100 data sets.

### 239 Results

240 The rRMSE values of the estimated parameters (fixed-ef-  
241 fect and random-effect) versus the level of IIV, stratified  
242 based on the different PK parameters, are shown in  
243 Fig. 2. The scale for each of the plots are adjusted to in-  
244 clude all values. The analysis of all parameter rRMSE  
245 values showed that all four tested estimation methods  
246 performed equally at the lower IIV levels (5–30%), while  
247 the performance degraded with an increase in IIV. The  
248 FOCE-I method performed better than the other three  
249 EM-based estimation methods; this was more apparent  
250 at higher IIV levels (above 30%) for both fixed-effect and  
251 random-effect parameters. Performance of both the



**Fig. 2** rRMSE plot for simulated data sets with 10%, 20%, 30%, 50% and 50% inter-individual variability. Relative root mean square error (rRMSE) of fixed-effect and random-effect parameters from simulated data sets with 10%, 20%, 30%, 50% and 50% inter-individual variability using FOCE-I (●), BAYES(C) (▲), BAYES(F) (■) and BAYES (+) estimation methods

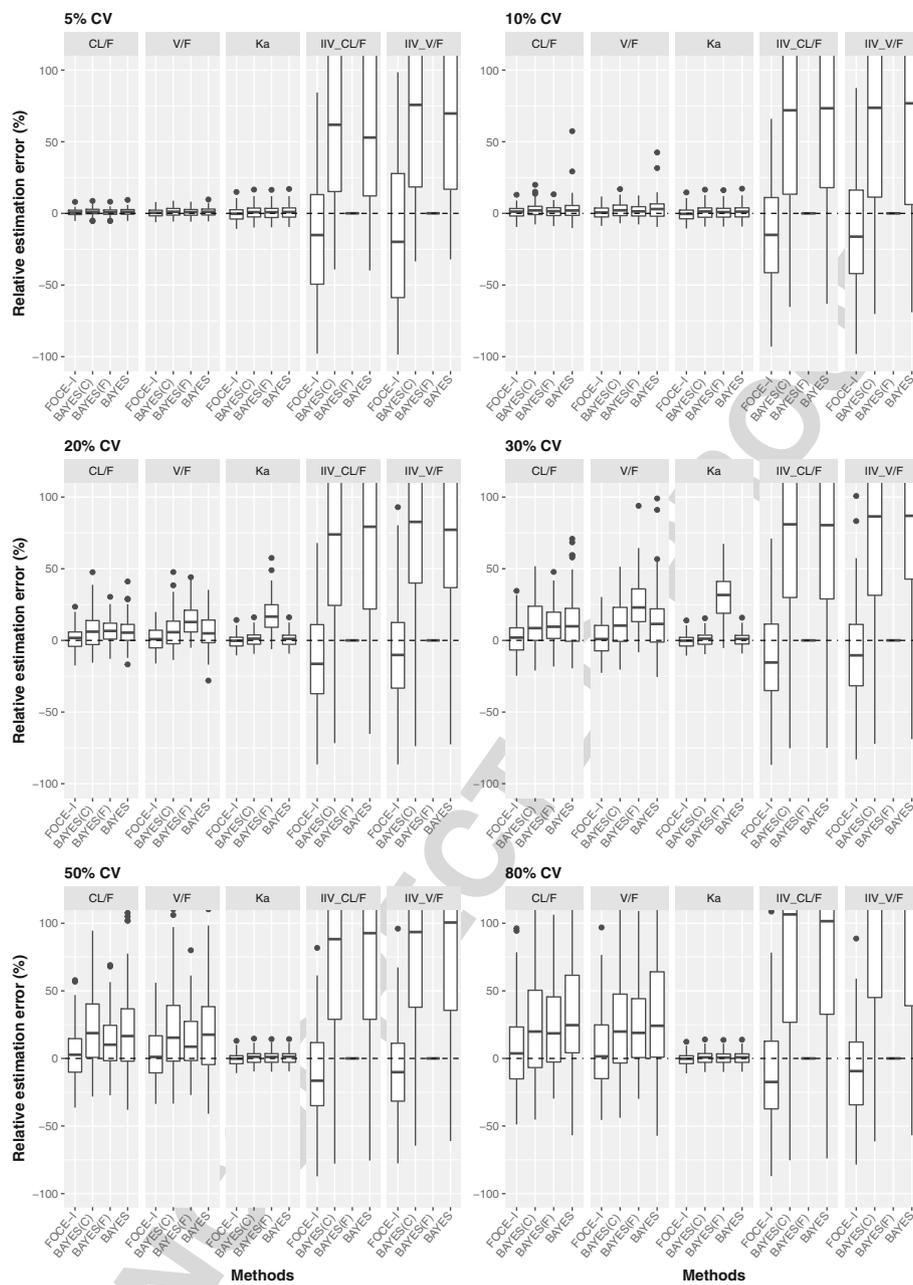
f2.1  
f2.2  
f2.3  
f2.4

251 BAYES(C) and BAYES methods were poor at an IIV  
252 greater than 30% in terms of rRMSE. All parameter estimates at 50% and 80% IIV had exceptionally high  
253 rRMSE. The BAYES(F) performance was intermediary  
254 between FOCE-I and BAYES(C)/BAYES estimation  
255 methods in terms of rRMSE.  
256

257 The REE of both fixed-effect and random-effect parameters versus the estimation methods, stratified by different levels of IIV, are shown in Fig. 3. The plots were  
258 adjusted to include  $\pm 100\%$  REE for the purpose of clarity. In general, all estimation methods overestimated  
259 fixed-effect parameters to some extent. At a lower level of IIV (5–10%), all estimation methods estimated fixed-effect  
260 parameters with negligible bias and reasonable  
261 precision. However, the bias as well as imprecision  
262  
263  
264  
265

266 increased with an increase in IIV variability. Overall, FOCE-I estimated fixed-effect parameters with REE near  
267 zero at all tested levels of IIV, while the distribution of REE increased with an increase in IIV. The other  
268 remaining three methods, BAYES(C), BAYES(F), and BAYES, had comparatively higher REE with a wider  
269 distribution range compared with the FOCE-I method.  
270  
271  
272

273 The estimation of random-effect parameters had pronounced bias and imprecision, irrespective of the estimation  
274 method used or the level of IIV (with the exception of the BAYES(F) method, where the variance parameter  
275 was fixed to the true value) as shown in Fig. 3. Both EM-based methods, BAYES(C) and BAYES, performed poorly  
276 with higher bias and impression. Across all tested levels of IIV, BAYES(C) and BAYES methods had high bias and  
277  
278  
279  
280



**Fig. 3** REE box-plot for simulated data sets with 10%, 20%, 30%, 50% and 50% inter-individual variability. Box-plot of relative estimation error (REE) of fixed-effect and random-effect parameters from simulated data sets with 10%, 20%, 30%, 50% and 50% inter-individual variability using FOCE-I, BAYES(C), BAYES(F) and BAYES estimation methods

f3.1  
f3.2  
Q4  
f3.3  
f3.4

281 precision with skewed distribution of positive REE. The  
282 FOCE-I method consistently performed better compared  
283 with other methods with much lower and slightly negative  
284 bias where the distribution of REE overlapped with the  
285 zero value.

286 The overall stability of estimations were high with a  
287 100% success rate of minimization and covariance step  
288 for BAYES(C), BAYES(F), and BAYES methods. For the  
289 FOCE-I method, the minimization step had a 100%

290 success rate, but the rate of the successful covariance  
291 step was 52% at 5% IIV while other estimations had a  
292 successful covariance step close to 100%.

293 The THEO data set used as a case study had 132  
294 observations from 12 subjects, 11 observations per indi-  
295 vidual after an oral dose of 320 mg theophylline. A one-  
296 compartment PK model with first order absorption  
297 described the data well and it was used as a final model.  
298 The PK parameters from the THEO data set were: *CL/F*

299 = 2.88 l/h,  $V_d/F = 33.01$  l and  $k_a = 1.46$  1/h and IIV were  
300 25.69%, 13.48% and 65.39%, respectively. The rRMSE  
301 plots (Additional file 1) of the PK parameters from the  
302 THEO model show that the performance of the four  
303 estimation methods were similar for estimates of  $CL/F$   
304 and  $V_d/F$  both of which had lower IIV, below 30%.  
305 Whereas, overall higher rRMSE for estimate of  $K_a$  was  
306 observed, particularly from EM based methods. The esti-  
307 mation methods followed similar pattern of performance  
308 as indicated by rRMSE for estimation of random effect  
309 parameters. Similarly, REE box plots (Additional file 2)  
310 for estimated PK parameters show that  $CL/F$  and  $V_d/F$   
311 estimated by all four estimation methods were very close  
312 to the true values, where both of them had true IIV  
313 below 30%. For the estimate of  $K_a$ , FOCE-I method esti-  
314 mated values were close to the true value while esti-  
315 mated values from other three EM based method were  
316 positively biased (median REE above 25%) with low pre-  
317 cision. Estimation of random effect parameters were  
318 poor for all the estimation methods, but the FOCE-I  
319 method performed relatively better in terms of bias and  
320 precision.

## 321 Discussion

322 For an estimation method, the most desirable features  
323 are a low bias and high precision. In this study, we used  
324 rRMSE and REE to evaluate these features. The rRMSE  
325 provides a single value that indicates both bias and pre-  
326 cision. Moreover, rRMSE provides a way to compare  
327 performance across parameters and models. However,  
328 the REE allows for comparison of different parameters  
329 with varying magnitudes in a single plot while acknow-  
330 ledging bias and precision. For an estimation method to  
331 be unbiased and precise, the REE should have a normal  
332 distribution with a median of 0 and a narrow range of  
333 values.

334 The FOCE-I method performed better among the four  
335 methods tested based on the overall rRMSE. This per-  
336 formance was supported by the REE plot, which did not  
337 show any significant bias for any fixed effect parameters  
338 at any given level of IIV. The median REE values for the  
339 random-effect parameters were not greater than -17% at  
340 any given level of IIV. A resembling result of negative  
341 bias was observed with the FOCE-I algorithm in a simi-  
342 lar studies comparing different estimation methods [9].  
343 The FOCE-I method has been shown to work suffi-  
344 ciently well for simple models when compared to other  
345 EM based algorithms in previous studies. Furthermore,  
346 when the IIV was low, the performance of classical esti-  
347 mation methods and EM based methods were very close.  
348 Similar results were observed in a previous study for  
349 such simple model (1-compartment model), where the  
350 performance of those estimation methods were found to  
351 be nearly equal [5].

On the other hand, rRMSE values for the three BAYES-  
based methods were significantly higher for both fixed-  
and random-effect parameters at higher levels of IIV. The  
higher rRMSEs were due to the wider spread of outliers,  
more so at higher levels of IIV. A similar trend of rRMSE  
of estimated parameters was observed using BAYES  
methods by Johansson et al., where the highly distorted  
rRMSE rendered the estimated parameters meaningless  
[9]. The performances of the BAYES-based methods were  
poor, with high bias and low precision. Even with the  
utilization of true values for all initial parameters, the  
BAYES(F) method was not able to estimate parameters  
close to the true values. Similarly, the median REE for all  
three methods based on the BAYES-method was compar-  
atively higher for fixed-effect parameters and signifi-  
cantly higher for random-effect parameters, compared  
with those of the classical FOCE-I method. There was also  
a general trend of an increase in REE (positive) with an  
increase in IIV. These observations with BAYES-based  
methods can be attributed to the way in which the BAYES  
method estimates the parameters i.e., by generating a large  
set of probable population parameters and variance  
parameters that represent the distribution according to  
their ability to fit the data [11]. Therefore, the limited  
number of subjects used in the study may be the reason  
for the poor performance of the three BAYES-based  
methods. However, a previous study showed that the  
BAYES method can provide robust estimates of complex  
PK/PD models with rich data and reliable priors [10].

The classical estimation method, FOCE-I, and max-  
imum likelihood EM-based BAYES method differ in  
their convergence criteria, where the former is based on  
changes in the parameter estimation gradient and are  
tested by default, and the latter is based on changes in  
objective function value and parameter estimates. The  
BAYES method can also define the convergence test  
type, and one can choose from no test, tests accessing  
changes in objective function, thetas and sigmas only,  
the addition of diagonals of omegas, or the addition of  
all omegas. For these reasons, the convergence rate was  
not included as a factor for comparison of estimation  
methods. However, all four methods tested at any level  
of IIV showed a 100% convergence rate. Additionally, in  
all estimation methods, the default or generally used  
values were used for options in the \$ESTIMATION  
block. It is possible to optimize the outcomes by chang-  
ing the values for different options in \$ESTIMATION  
block [4]. However, this aspect of the estimation method  
was not compared, as this study only explored the prac-  
tice of most users.

In this study, FOCE-I, the classical method, performed  
better with lower bias and higher precision compared  
with other BAYES-based methods. Moreover, the FOCE-  
I method is known to have a shorter run time than any

other new methods [9, 12]. The work presented here compares a classical estimation method, FOCE-I, and BAYES method, with different options in the \$ESTIMATION block and fixed OMEGA values (BAYES(C), BAYES(F), and BAYES) for population analysis of data with a low number of subjects ( $n = 6$ ). Moreover, the models built had only one compartment, with basic PK parameters and random effects on two PK parameters. Therefore, it should be noted that the structure and complexity of a model might vary (increase or decrease) in different pharmacometric projects or within the same project from the initial to final step. In contrast to our study, other studies have shown that for complex models with highly non-linear functions [12], highly skewed count distributions [13, 14], and/or low variability or very rare events [15], the classical methods exhibit marked bias and impression. Additionally, the selection of an estimation method for a particular modeling project can depend on various aspects including bias, precision, robustness, runtime, data type, timeframe of project, application of results etc. which are objective in nature as well as subjective aspect such as preference for particular estimation method based on knowledge and previous experience. Ultimately, a pharmacometrician needs to make a choice for an estimation method based on multiple aspects.

The data sets used in this study, unlike real clinical data, were simulated. IIV for all PK parameters were assumed to be the same for an individual; i.e., IIV was either 5%, 10%, 20%, 30%, 50%, or 80% for  $K_a$ ,  $CL/F$ , and  $V_d/F$ . In clinical scenarios, the CV may vary widely among the PK parameters within an individual. Therefore, to assess the relevance of results obtained from simulated data, a clinical data of theophylline involving 12 subjects, THEO data set, was used as a case study. The limitation of using real data is that the expected true parameters value is unknown. So, SSE was performed, where the parameter estimates from final THEO model was considered to be true parameters. And the parameter estimates from different estimation methods were compared to so-called true values for compare their performance. Similarity in the performance of all four estimations methods at lower IIV and better performance of FOCE-I methods at higher IIV was demonstrated by the rRMSE and REE of the estimated parameters. This further supports the results from the simulated data. Another limitation of this study is that only FOCE-I and BAYES methods were tested and compared. To further explore the best estimation method when dealing with a low number of subjects, other methods in NONMEM, such as LAPLACE, ITS, IMP, IMPMAP, and SAEM should also be evaluated in future studies.

## Conclusions

The FOCE-I, a classical estimation method, yielded better results in terms of bias and precision across all levels of IIV in comparison to three variations of BAYES estimation methods. The difference in performance between FOCE-I and three BAYES estimation methods in estimating fixed-effect parameters were significant only at the IIV level greater than 30%. The bias and imprecision of random-effect parameters were higher compared with fixed-effect parameters, however, it was consistently lower for FOCE-I method compared to those estimated using BAYES(C) and BAYES methods. These results were further supported by the results from the THEO data, where clinical data was used to simultaneously simulate and estimate PK parameters using FOCE-I and three BAYES estimation methods.

In conclusion, the classical FOCE-I method estimated the PK parameters more reliably than the BAYES method when using a simple model and data containing only a few subjects. After the base modeling step is complete and/or at the pivotal modeling step, use of other EM-based estimation methods can be considered for adapting to specific needs of the project.

## Additional files

**Additional file 1:** rRMSE plot for THEO data set. Relative root mean square error (rRMSE) of fixed-effect and random-effect parameters from THEO data set using FOCE-I (●), BAYES(C) (▲), BAYES(F) (■) and BAYES (+) estimation methods. (PDF 6 kb)

**Additional file 2:** REE box for THEO data set. Box-plot of relative estimation error (REE) of fixed-effect and random-effect parameters from THEO data set using FOCE-I, BAYES(C), BAYES(F) and BAYES estimation methods. (PDF 8 kb)

## Abbreviations

BAYES: Markov chain Monte Carlo Bayesian; BAYES(C): First-order conditional estimation with interaction and Markov chain Monte Carlo Bayesian composite method; BAYES(F): Markov chain Monte Carlo Bayesian with variance parameter fixed to true value; CL/F: Apparent clearance;  $\mathcal{E}$ : Residual variability; EM: Expectation maximisation; FOCE: First-order conditional estimation; FOCE-I: First-order conditional estimation with interaction; IIV: Inter-individual variability; IMP: Important sampling; IMPMAP: Important sampling assisted by mode a posteriori; ITS: Iterative two-stage;  $K_a$ : Absorption rate constant;  $\eta$ : Inter-individual variability; PK: Pharmacokinetic; REE: Relative estimation error; rRMSE: Relative root mean square error; SAEM: Stochastic approximation expectation maximization; SSE: Stochastic simulation and estimation;  $V_d/F$ : Apparent volume of distribution

## Acknowledgements

Not applicable.

## Funding

This research was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF), funded by the Ministry of Science, ICT & Future Planning (Grant 2009-0093815 and 2014R1A1A1006006) and supported by a grant of the Korea Health Technology R&D Project through the Korea Health Industry Development Institute (KHIDI), funded by the Ministry of Health & Welfare, Republic of Korea (Grand number: HI17C0927). This work was also supported by research funds from Chungnam National University.

**519 Availability of data and materials**

520 Data used in this study was simulated in NONMEM using SSE tool. THEO  
521 data used as a case study is available in the NONMEM distribution media.

**522 Authors' contributions**

523 SP and BS made substantial contributions to conception and design of the  
524 study and performed the simulation and estimation part with assistance  
525 from JL. SP and BS drafted the manuscript. JC, KIK, HB and NH assisted in  
526 critical analysis and interpretation of data as well as revising the contents  
527 critically for every draft of the manuscript. This work was performed under  
528 direct supervision and guidance of KK and HY, they guided the team from  
529 initial concept of the project through many iterative modifications in  
530 methods, data analysis and interpretation, critically revising every draft of  
531 manuscript and approving final version to be submitted. All authors agree to  
532 be accountable for all aspects of the work in ensuing the questions related  
533 to the accuracy or integrity of any part of the work are appropriately  
534 investigated and resolved.

**535 Ethics approval and consent to participate**

536 Not applicable.

**537 Consent for publication**

538 Not applicable.

**539 Competing interests**

540 The authors declare that they have no competing interests.

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552 Received: 25 January 2017 Accepted: 15 November 2017

553 Published online: 01 December 2017

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